Math 10 Lecture Videos

Section 1.2

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OBJECTIVES:

1. Convert between mixed numbers and improper fractions.
2. Write the prime factorization of a composite number.
3. Reduce or simplify fractions.
4. Multiply fractions.
5. Divide fractions.
6. Add and subtract fractions with identical denominators.
7. Add and subtract fractions with unlike denominators.

Objective 1: Convert between mixed numbers and improper fractions

Converting a Mixed Number to an Improper Fraction
1. Multiply the denominator of the fraction by the natural number and add the numerator to this product.
2. Place the result from step 1 over the denominator in the original mixed number.

Convert $2\frac{3}{4}$ **to an improper fraction.**

$$2\frac{3}{4} = \frac{4 \cdot 2 + 3}{4} = \frac{11}{4}$$

Mixed number $\rightarrow$ Improper fraction

Objective 1: Convert between mixed numbers and improper fractions

Converting an Improper Fraction to a Mixed Number
1.   Divide the denominator into the numerator. Record the quotient and the remainder.
2.   Write the mixed number using the form: $\text{quotient} \dfrac{\text{remainder}}{\text{original denominator}}$.

Convert $\dfrac{15}{2}$ to a mixed number.

$\dfrac{15}{2}$ 15 divided by 2 yields 7 with a remainder of 1

$\dfrac{15}{2} = 7\dfrac{1}{2}$

Improper fraction → Mixed number

Objective 2: **Write the prime factorization of a composite number**

- A composite number is a natural number greater than 1 that is not prime.

- Every composite number can be expressed as the product of prime numbers.

- Examples:

$$90 = 2 \cdot 3 \cdot 3 \cdot 5 \qquad 36 = 4 \cdot 9$$

$$= 2 \cdot 2 \cdot 3 \cdot 3$$

Objective 3:
Reduce or simplify fractions

Reducing a Fraction to Its Lowest Terms
1. Write the prime factorizations of the numerator and the denominator.
2. Divide the numerator and the denominator by the greatest common factor, the product of all factors common to both.

Reduce the following fractions to its lowest terms.

$$\frac{15}{27} = \frac{3 \cdot 5}{3 \cdot 3 \cdot 3} = \frac{5}{3 \cdot 3} = \frac{5}{9}$$

$$\frac{10}{15} = \frac{2 \cdot \cancel{5}}{3 \cdot \cancel{5}} = \frac{2}{3}$$

$$\frac{13}{15} = ?$$

Objective 4: Multiplying Fractions

The product of two or more fractions is the product of their numerators divided by the product of their denominators.

In algebraic language: If $\dfrac{a}{b}$ and $\dfrac{c}{d}$ are fractions, then $\dfrac{a}{b} \cdot \dfrac{c}{d} = \dfrac{a \cdot c}{b \cdot d}$.

Examples:

Multiply $\dfrac{4}{11} \cdot \dfrac{2}{3}$.

$$\dfrac{4}{11} \cdot \dfrac{2}{3} = \dfrac{4 \cdot 2}{11 \cdot 3}$$
$$= \dfrac{8}{33}$$

Multiply $\left(3\dfrac{2}{5} \right)\left(1\dfrac{1}{2} \right)$.

$$\left(3\dfrac{2}{5} \right)\left(1\dfrac{1}{2} \right) = \dfrac{17}{5} \cdot \dfrac{3}{2}$$
$$= \dfrac{51}{10}$$
$$= 5\dfrac{1}{10}$$

Objective 5: Dividing Fractions

The quotient of two fractions is the first fraction multiplied by the reciprocal of the second fraction.

In algebraic language:
$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}.$$

Replace $\frac{c}{d}$ with its reciprocal by interchanging its numerator and denominator.

Change division to multiplication.

Objective 5: **Dividing Fractions**

Examples:

Divide $\dfrac{5}{4} \div \dfrac{3}{8}$.

$$\dfrac{5}{4} \div \dfrac{3}{8} = \dfrac{5}{4} \cdot \dfrac{8}{3}$$

$$= \dfrac{5}{\cancel{4}} \cdot \dfrac{\cancel{4} \cdot 2}{3}$$

$$= \dfrac{10}{3}$$

$$= 3\dfrac{1}{3}$$

Divide $3\dfrac{3}{8} \div 2\dfrac{1}{4}$.

$$3\dfrac{3}{8} \div 2\dfrac{1}{4} = \dfrac{27}{8} \div \dfrac{9}{4}$$

$$= \dfrac{27}{8} \cdot \dfrac{4}{9}$$

$$= \dfrac{\cancel{9} \cdot 3}{\cancel{4} \cdot 2} \cdot \dfrac{\cancel{4}}{\cancel{9}}$$

$$= \dfrac{3}{2} = 1\dfrac{1}{2}$$

Objective 6: Add and subtract fractions with identical denominators.

- When adding or subtracting fractions, it is necessary to rewrite the fractions as fractions having the same denominator, which is called the **common denominator** for the fractions being combined.

- The sum or difference of two fractions with identical denominators is the sum or difference of their numerators over the common denominator.

- In algebraic language: If $\dfrac{a}{b}$ and $\dfrac{c}{d}$ are fractions, then $\dfrac{a}{b} + \dfrac{c}{b} = \dfrac{a+c}{b}$ and $\dfrac{a}{b} - \dfrac{c}{b} = \dfrac{a-c}{b}$.

Objective 6: Add and subtract fractions with identical denominators.

Examples:

Perform the indicated operation : $\dfrac{2}{11} + \dfrac{3}{11}$

$$\dfrac{2}{11} + \dfrac{3}{11} = \dfrac{2+3}{11}$$

$$= \dfrac{5}{11}$$

Perform the indicated operation : $\dfrac{5}{6} - \dfrac{1}{6}$

$$\dfrac{5}{6} - \dfrac{1}{6} = \dfrac{4}{6}$$

$$= \dfrac{2}{3}$$

Objective 7: Add and subtract fractions with unlike denominators

Adding and Subtracting Fractions with Unlike Denominators
1.   Rewrite the fractions as equivalent fractions with the least common denominator.
2.   Add or subtract the numerators, putting this result over the common denominator.

Finding the Least Common Denominator:

1. Find the prime factor for each denominator.
2. The least common denominator is found by using each factor the greatest number of times it appears in any denominator.

Objective 7: Add and subtract fractions with unlike denominators

Add: $\dfrac{2}{15} + \dfrac{5}{9}$

① Find the lowest common denominator by using the prime factorization of each denominator.

$15 = 3 \cdot 5$

$9 = 3 \cdot 3$

② The least common denominator is found by using each factor the greatest number of times it appears in any denominator.

$(5)(3)(3) = 45$

③ $\dfrac{2}{15} \cdot \dfrac{3}{3} + \dfrac{5}{9} \cdot \dfrac{5}{5} = \dfrac{6}{45} + \dfrac{25}{45} = \dfrac{31}{45}$

Objective 7: Add and subtract fractions with unlike denominators

Perform the indicated operation: $3\dfrac{1}{6} - 1\dfrac{11}{12}$

$$3\dfrac{1}{6} - 1\dfrac{11}{12} = \dfrac{19}{6} - \dfrac{23}{12}$$

$$= \dfrac{19 \cdot 2}{6 \cdot 2} - \dfrac{23}{12}$$

$$= \dfrac{38}{12} - \dfrac{23}{12}$$

$$= \dfrac{15}{12} = \dfrac{5}{4} = 1\dfrac{1}{4}$$

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1. Convert between mixed numbers and improper fractions. ✓
2. Write the prime factorization of a composite number. ✓
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4. Multiply fractions. ✓
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6. Add and subtract fractions with identical denominators. ✓
7. Add and subtract fractions with unlike denominators. ✓